

# Design of Hydrogen Maser Cavity Tuning Servo

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*Servo design details are described for a prototype hydrogen maser cavity tuner for use with the hydrogen maser frequency standards developed at the Jet Propulsion Laboratory.*

This article is a continuation of a tuner design discussion initiated in Ref. 1.

A period counter forms the basis of the tuning system. It has a sampled transfer characteristic given by

$$\tau_{\text{out}} = \frac{1}{s} \left( \tau_0 + \sum_{n=0}^{\infty} \tau_n e^{-s\tau n} \right) \quad (1)$$

where

$\tau_{\text{out}}$  = output count

$\tau_n$  = input variable at sampling time  $N\tau$

$\tau_0$  = initial value

The counter will perform a summation of difference measurements if the count direction is reversed after each measurement, and the count is sampled after alternate measurements:

$$\tau_{\text{out}} = \frac{1}{s} \left\{ \tau_0 + \sum_{n=0}^{\infty} (\tau_{2n} - \tau_{2n+1}) \right\} e^{-2s\tau n} \quad (2)$$

Where it is desired to remove a linear temporal change or drift in the input parameter, the count direction may be double switched to give

$$\tau_{\text{out}} = \frac{1}{s} \left\{ \tau_0 + \sum_{n=0}^{\infty} (\tau_{4n+1} - \tau_{4n+2} - \tau_{4n+3} + \tau_{4n+4}) \right\} e^{-2s\tau n} \quad (3)$$

Alternate adjacent samples are subtracted and the differences are accumulated in a buffer which samples the counter at a  $2\tau$  sampling rate. A diagram of the complete servo is shown in Fig. 1. The maser section of Fig. 1 represents the cavity pulling relationship described previously (Ref. 1). The output frequency  $f_o$  of the maser is modulated by switching between two cavity pulling factors  $K_1$  and  $K_2$ . The switch sequence is that of Eq. (3). Therefore, changes in the maser output frequency will be coherently summed in the up-down counter of the tuner, while static and linear changes in the maser frequency will produce zero and zero-time-average counts in the tuner, respectively.

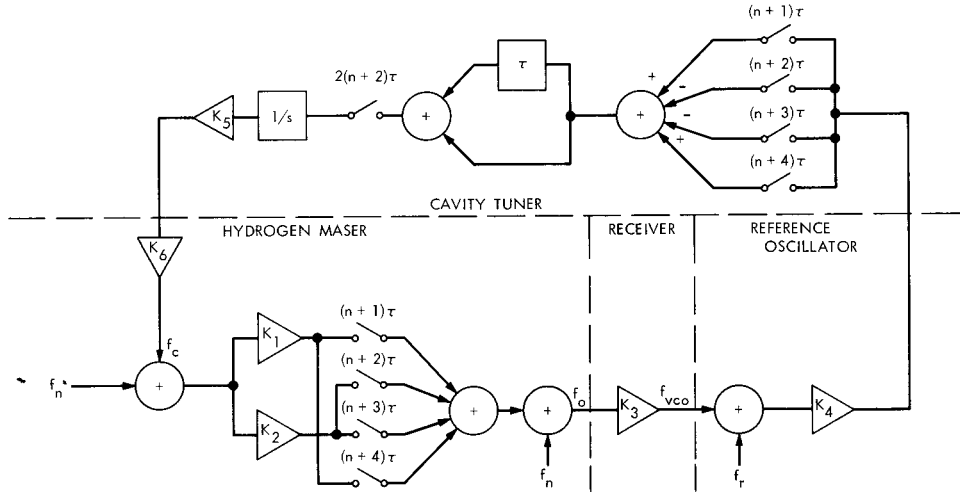


Fig. 1. Cavity tuner servo diagram

The receiver divides the maser frequency,  $f_o$ , by the ratio ( $K_3$ ) between the maser (1420.405 MHz) and the voltage-controlled oscillator (VCO) (100 MHz) of the receiver. The 100-MHz VCO is then mixed with the reference oscillator  $f_r$  to produce a nominal 100-s beat period  $\tau_b$ . This period is measured in the tuner by counting a 1-kHz internal clock for the interval between zero crossings of  $\tau_b$ . The proportionality constant  $K_4$  between the frequency and period variable is given by

$$K_4 = \frac{\delta \tau_b}{\delta f_b} = \tau_0^2$$

where  $\tau_b$  average is much greater than modulation or noise produced on  $\tau_b$ .

In the count sequence, the counter counts up for the first period, allows the dissociator pressure to reach equilibrium for one period, then counts down the next. After completion of the down count, the residual in the counter is transferred to a digital buffer and an analog-to-digital converter. The voltage from the analog-to-digital converter in turn is applied to a varactor which tunes the microwave cavity. The sequence is then inverted with the down count coming first.  $K_5$  is derived from the counter clock rate, the analog amplifier of the digital-to-analog converter, and the position of the counter digits selected for analog conversion. The last gain constant  $K_6$  is the sensitivity of the tuning varactor to its control voltage from the digital-to-analog converter.

The open-loop transfer function for the system becomes

$$\frac{f_c}{f_n} = \frac{(K_1 - K_2) K_3 K_4 K_5 K_6}{s} e^{-2s\tau} = \frac{\alpha e^{-2s\tau}}{s}$$

where  $\alpha$  is the loop gain. The closed-loop transfer function (sampled) becomes

$$\frac{f_c}{f_n} = \frac{\alpha e^{-2s\tau}}{1 - (1 - \alpha) e^{-2s\tau}}$$

and the response to the input step (sampled) is

$$f_c = f_{co} \left\{ \sum_{n=0}^{\infty} [1 - (1 - \alpha)^n] e^{-2s\tau(n+1)} \right\}$$

The convergence of the loop is, therefore, a linear decreasing staircase function. Figure 2 is a graph of the loop error as a function of time and loop gain.

A simplified design criteria is given by

$$\sigma_1 \approx \alpha \sigma_2 \geq \epsilon$$

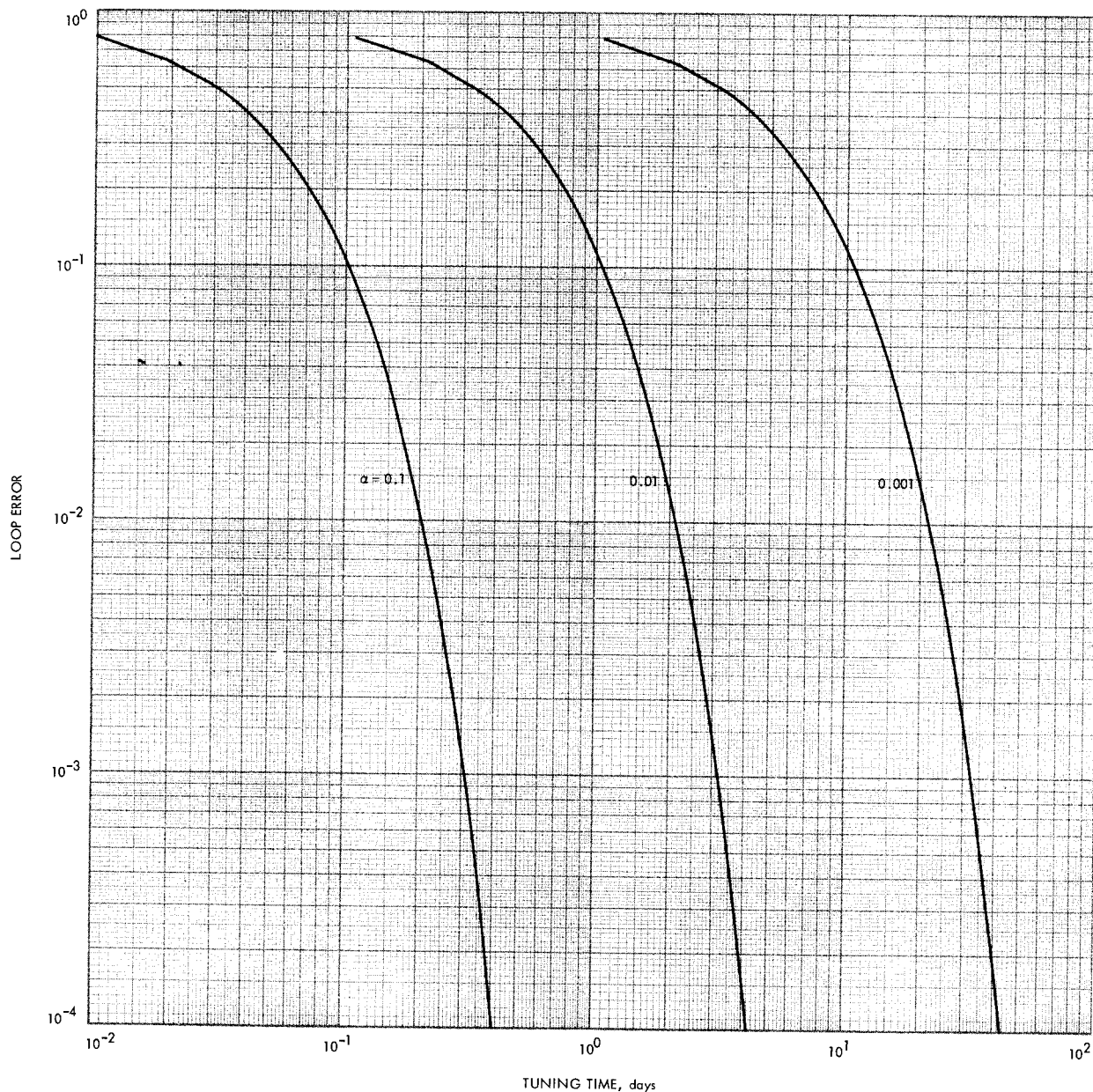
where

$\sigma_1$  = allowable variation of maser long-term stability

$\sigma_2$  = measurement variation of  $\tau_b$

$\epsilon$  = change in maser output frequency for the least significant bit of the digital-to-analog converter

The loop gain is approximately equal to the input measurement of  $\tau_b$  transferred to the output frequency. Hence, if the reference oscillator is one hundred times



**Fig. 2. Tuning error vs tuning time and loop gain**

more noisy ( $\sigma_f/f$ ) than the hydrogen maser, a degradation in tuning accuracy must be accepted or tuning times on the order of 40 days are required to realize the inherent

accuracy of the maser. The above assumes the noise introduced into the maser output by the tuner to be 10% of the maser 100-s stability.

## Reference

1. Finnie, C., "Frequency Generation and Control: Atomic Hydrogen Maser Frequency Standard," in *The Deep Space Network Progress Report*, Technical Report 32-1526, Vol. I, pp. 73-75. Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1971.